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If examination is limited to crystals of visible dimensions, then the surface separating the new and old phases may be considered as having a slight curvature. In the first approximation, we disregard the influence of curvature and we assume that the equilibrium carbon concentration on the external side of the crystal's surface agrees with the value given in the diagram describing the equilibrium between iron and carbon when the GS line is extrapolated into the region of subcritical temperatures [2]:

$$c_{\text{equi}} = 0.8 + 0.013(723 - T) \quad (2)$$

where  $T$  is the temperature at which the ferrite crystal begins to grow.

Condition (1) can be fulfilled only by equating (a) the amount of carbon separating out on the crystal surface, during an infinitesimal moment of time  $dt$  and outgoing in this same interval of time due to diffusion, to (b) the austenite not yet converted. The equation describing the conservation (balance) of mass on the boundary of the growing crystal can thus be written:

$$(c_{\text{equi}} - c_{n.ph}) \frac{d\rho(t)}{dt} = -D \left( \frac{\partial c}{\partial r} \right)_{r=\rho(t)}, \quad (3)$$

where  $c_{n.ph}$  is the carbon concentration in the new phase ( $n.ph$ ) and  $D$  the coefficient of carbon diffusion in austenite.

We will assume that at the stage of ferrite-crystalline growth with which we are concerned the nuclei of the new phase grow independently of one another; that is,

$$\text{if} \quad c(\infty, t) = c_0, \quad (4)$$

$$c(r, 0) = c_0, \quad r > 0, \quad (5)$$

where  $C_0$  is the original carbon concentration in the austenite.

The process of carbon diffusion in austenite is governed by the equation

$$\frac{\partial^2 c}{\partial r^2} + \frac{2}{r} \frac{\partial c}{\partial r} = \frac{1}{D} \frac{\partial c}{\partial t}. \quad (6)$$

The solution of equation (6) under conditions (1), (3), (4), and (5) can be affected, for example, by the method suggested by G. P. Ivantsov [3] (see Figure 1). It permits determining  $c(r, t)$  for  $t > 0$ ,  $r > \rho(t)$  and  $\rho(t)$ . Without going into the various steps, the final result will be:

$$c(r, t) = c_0 + (c_{\text{equi}} - c_0) \frac{\frac{2\sqrt{D\tau}}{r} e^{-r^2/4D\tau} - \sqrt{\pi} \operatorname{erfc}\left(\frac{r}{2\sqrt{D\tau}}\right)}{\frac{1}{\beta} e^{-\beta^2} - \sqrt{\pi} \operatorname{erfc}(\beta)}, \quad (7)$$

$$\text{where } \operatorname{erfc}(\beta) = \frac{2}{\sqrt{\pi}} \int_{\beta}^{\infty} e^{-\xi^2} d\xi; \quad \rho(t) = 2\beta\sqrt{D\tau}, \quad (8)$$

$\beta$  is the root of the transcendental equation:

$$\frac{c_{\text{equi}} - c_0}{c_{\text{equi}} - c_{n.ph}} = 2\beta^2 [1 - \beta\sqrt{\pi} e^{\beta^2} \operatorname{erfc}(\beta)] = F(\beta). \quad (9)$$

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The function of  $F(\beta)$  for  $0 \leq \beta \leq 2$  is shown in Figure 1.

Thus, the speed of growth of a ferrite grain is

$$v(t) = \frac{dp(t)}{dt} = \frac{\theta \sqrt{D}}{\sqrt{F}}, \quad (10)$$

where

$$\log D = -3.00 - 6.82 \left( \frac{1}{273 + T} - 5.8 \cdot 10^{-4} \right) \cdot 10^3, \quad (11)$$

if  $D$  is expressed in square millimeters per second [2].

For example, let

$$c_0 = 0.49, \quad c_{nph} = 0.09, \quad T = 720^\circ \text{C};$$

then

$$\frac{c_{equi} - c_0}{c_{equi} - c_{nph}} \cong 0.4.$$

From Figure 1 we find that

$$F(0.8) \cong 0.4; \text{ therefore } \beta \cong 0.8.$$

In this case

$$v \cong \frac{240 \cdot 10^{-6}}{\sqrt{F}} \text{ mm/sec} \quad (12)$$

In Figure 2, the solid curve is derived from formula (12) and the dotted curve is determined on the basis of experiments performed by V. E. Neymark and R. I. Entin with the help of I. B. Piletska.

The completely satisfactory agreement between theory and experimental data indicates the accuracy of our initial assumptions.

The scheme set forth above may be applied to the calculation of the rate of growth of crystalline grains of the new phase and for other metallic systems.

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## BIBLIOGRAPHY

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2. W. H. Brandt, J. Applied Physics, 16, 139 (1945)
3. G. P. Ivantsov, DAN, 58, 237, (1947)

[Appendix figures follow.]

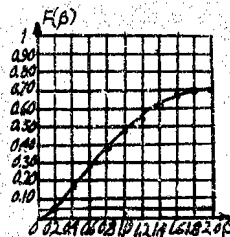


Figure 1

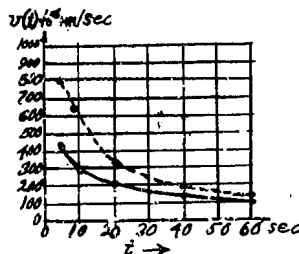


Figure 2

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